WILLIAM R. EASTERLY
PAOLO MAURO
KLAUS SCHMIDT-HEBBEL

# Money Demand and Seigniorage-Maximizing Inflation

THERE IS WIDESPREAD CONSENSUS among economists that high inflation is often caused by the government's need to raise seigniorage in order to finance high budget deficits (Sargent 1982; Dornbusch and Fischer 1986; Van Wijnbergen 1989; Buiter 1990; and Easterly and Schmidt-Hebbel 1994).[1] Depending on the shape of the money demand function, steady-state seigniorage may follow a Laffer curve, where seigniorage first rises and then falls with higher inflation. If this is the case, then there exists a rate of inflation that maximizes steady-state seigniorage, $\pi_{max}$. Knowledge of $\pi_{max}$ may be of interest both in order to discriminate between existing theories of inflation and for policy purposes.

The literature has put forward various hypotheses to explain high inflation. First, the economy may be experiencing high inflation, but still be on the left-hand side of the Laffer curve. In this case, inflation may be seen as a result of the government's need to raise seigniorage. Second, the economy may be on the "wrong" side of the Laffer curve. If inflation is above $\pi_{max}$ and is fairly stable, then this may be inter-

The authors are indebted to Robert Barro, Alex Cukierman, José De Gregorio, Fernando de Holanda Barbosa, Ibrahim Elbadawi, Graham Elliott, Alan Gelb, Miguel Kiguel, Gregory Mankiw, Luis Serven, Raimundo Soto, Luis Suárez, Salvador Valdés-Prieto, Steve Webb, John Welch, two anonymous referees, and seminar participants at Harvard University, the XIth Latin American Meetings of the Econometric Society (Mexico) and the World Bank for detailed comments and discussions. Many thanks go also to Jorge Canales, Mauricio Carrizosa, Janvier Kporou-Litze, Daniel Oks, Jim Stephens, and Luis Suárez for providing some of the data for this paper.

1. There is an unfortunate lack of standard language in the literature that requires us to define our terms precisely: by seigniorage, we mean the total revenue from money creation, which includes two components: (1) the "inflation tax" which is the inflation rate times real money balances, and (2) the growth in real money balances. This terminology is the same as that followed by, for example, Blanchard and Fischer (1990). Some authors use "seigniorage" to refer to only the second component.

WILLIAM R. EASTERLY *and* KLAUS SCHMIDT-HEBBEL *are in the Policy Research Department at the World Bank.* PAOLO MAURO *is an economist at the International Monetary Fund.*

*Journal of Money, Credit, and Banking*, Vol. 27, No. 2 (May 1995)

preted as evidence in favor of the existence of a "high-inflation equilibrium." The latter may be a consequence of the absence of a nominal anchor, as in Bruno and Fischer (1990), or because of Barro-Gordon (1983) effects due to the inability of the monetary authorities to undertake credible commitments, as in Kiguel and Liviatan (1991), or because of the authorities' high discount rate that biases policy against stabilizations with short-term costs and long-term benefits. If inflation is above $\pi_{max}$ and is accelerating, this may be seen as the consequence of the government's need to raise seigniorage $s'$ in excess of that which it can obtain by setting inflation at $\pi_{max}$, $s(\pi_{max})$. $s'$ in excess of $s(\pi_{max})$ can only be collected by bringing about ever-accelerating inflation, as emphasized by Kiguel (1989).[2] In fact, $\pi$ higher than $\pi_{max}$ has been suggested by Rodriguez (1994) as a definition of a hyperinflation.

Estimates of the seigniorage-maximizing inflation rate may help to decide among such competing theories. Conventional estimates of the seigniorage-maximizing inflation rate generally make use of the Cagan (1956) function, where the log of real money is a linear function of inflation. The Cagan function is appealing because of its simplicity and its attractive algebraic property: $\pi_{max}$ is given by one over the coefficient on inflation. However, this simplicity is achieved at the expense of a restrictive functional form, which assumes a constant semielasticity of money demand with respect to inflation. The estimates of $\pi_{max}$ using the Cagan form also usually define the inflation rate affecting money demand as the percent change in prices (or sometimes the log change in prices), when theory implies that the correct opportunity cost of holding money per period is the inflation rate divided by one plus the inflation rate. Our aim is to test the sensitivity of estimates of $\pi_{max}$ to the assumption of constant semielasticity in the Cagan form and to the definition of the opportunity cost of money.

Section 1 develops a model of money demand, inflation, and seigniorage based on an optimizing consumer-investor-portfolio allocator, who faces a cash-in-advance constraint, forcing this agent to hold a combination of money and bonds before incurring consumption expenditure. It will be shown that the existence of a Laffer curve depends critically on how good substitutes money and bonds are in aggregate financial assets held as "cash"-in-advance.

Section 2 presents both individual-country and combined cross-country time-series evidence that supports the notion that the semielasticity of money demand with respect to inflation varies with inflation. The empirical evidence, collected for all high-inflation countries during the 1960–1990 period, is also used to provide estimates of the seigniorage-maximizing inflation rate. It will be shown that both these estimates and those for the semielasticity differ substantially from those obtained under the conventional Cagan approach. The results also have striking implications for the substitutability of bonds and money. The results will also show that the semielasticity and the seigniorage-maximizing inflation rate, when based on the correct measure of the opportunity cost of holding money, differ markedly from

2. Bruno and Fischer (1990) also mention this possibility. With reference to the hyperinflation of the 1920s, Barro (1972) notices that inflation tends to accelerate when the revenue-maximizing rate is exceeded.

those obtained when using conventional but incorrect measures of inflation. Section 3 concludes.

## 1. THE MODEL

This section develops a simple model of money demand, inflation, and seigniorage. It shows that both the semielasticity of money demand with respect to the opportunity cost of holding money and the inflation rate that maximizes seigniorage in the steady state depend on the degree of substitutability between money and bonds. In addition, the inflation semielasticity of money demand is shown to vary with inflation.

An infinitely lived optimizing representative agent takes consumption, investment, and portfolio decisions in a closed economy. This agent holds money and bonds for transaction purposes and maximizes a standard intertemporal utility function:

$$\max \int_0^{\infty} e^{-\rho t} \frac{c^{1-\sigma} - 1}{1 - \sigma} dt \tag{1}$$

where $\rho$ is the discount rate, $c$ is consumption, and $1/\sigma$ is the intertemporal elasticity of substitution.

Production ($y$) in the one-good economy is assumed to depend only on a broad concept of capital ($k$), as in Barro (1990) and Rebelo (1991):[3]

$$y = A\,k\,. \tag{2}$$

There are three assets available to the consumer—capital $k$, nonindexed money (real value $m$), and indexed money (real value $b$, referred to as "bonds" for short). Bonds pay no interest, but are fully indexed to the price level.[4] We assume that inflation cannot fall below $-A$. There is no uncertainty. Since capital has real return $A$ (net of depreciation), it always dominates bonds and money. However, a cash-in-advance constraint requires that some combination of money and bonds must be held in order to purchase consumption goods:

$$f(m, b) - c \geq 0 \tag{3}$$

3. Population is assumed fixed and normalized at one, so all variables can be interpreted in per capita terms.

4. The classic example of this kind of asset in developing countries is foreign currency, which maintains its value as the nominal exchange rate moves with the domestic price level. Other kinds of highly liquid financial assets often pay a nominal return adequate to compensate for inflation but little or no real return. Formally inflation-indexed assets paying a zero real return exist in some developing countries. This was the case of selected bank deposits held by households in China between mid-1988 and late 1991. UPAC deposits held in Colombia's savings and loan associations are highly liquid deposits indexed by consumer prices. Real assets are also often used as inflation hedges.

where $f$ is linearly homogeneous in $m$ and $b$, and satisfies $f_m > 0, f_b > 0, f_{mm} < 0$, $f_{bb} < 0, f_m(0, b) = \infty$ and $f_b(m, 0) = \infty$.[5] The last two conditions ensure that there are no corner solutions; the consumer always holds a positive amount of both money and bonds, which in general are imperfect substitutes.

This cash-in-advance constraint says that either money or bonds can be used for transactions.[6] This approach is in the same spirit as the Lucas and Stokey (1987) generalization of cash-in-advance models to include "cash" and "credit" goods. The intuitive justification is also similar to the "shopping costs" approach of Arrau and de Gregorio (1991).[7] An alternative approach is to assume that money provides utility. This option, followed by many studies, has been adopted recently by Calvo and Leiderman (1992) in deriving a variable semielasticity of money demand with regard to the opportunity cost of holding money.

The consumer faces the following budget constraint each period:

$$c = y + tr - I_m - I_b - I_k \tag{4}$$

where $tr$ are the real resources transferred from the government back to consumers in lump-sum form, and $I_m$, $I_b$, and $I_k$ are real flows of resources devoted to accumulation of money, bonds, and capital, respectively.

Bonds $b$ and money $m$ are the liabilities of the government. The government transfers exhaust the resources captured from consumers by issuing money and bonds:

$$tr = I_m + I_b\,. \tag{5}$$

We assume the government does not hold any other assets. Because our interest is in the steady-state equilibrium, we also rule out open market operations by the government (exchange of $b$ for $m$); changes in either money or bonds are assumed to occur *only* to finance transfers.

Asset accumulation is given by

$$\dot{m} = I_m - \pi m \tag{6}$$

$$\dot{b} = I_b \tag{7}$$

$$\dot{k} = I_k \tag{8}$$

where $\pi$ is the inflation rate. Dots denote time derivatives.

5. These conditions are all satisfied by the CES function introduced below.

6. A similar cash-in-advance constraint appears in Walsh (1984).

7. In high-inflation countries it is often necessary to pay in foreign currency when purchasing a house, though smaller transactions typically require the use of local currency. [See Calvo and Végh (1992) on currency substitution in developing countries.] This case is not fully captured by this model with one homogeneous good, but may help to develop intuition for the constraint in (3).

The consumer-producer solves the intertemporal problem (1)–(4) and (6)–(8) with perfect foresight. The first-order conditions imply the following standard expression (see Rebelo 1991 and Barro 1990) for the consumption (and output) growth rate $g$:[8]

$$g = (A - \rho)/\sigma \ . \tag{9}$$

Note that growth is not affected by inflation, which is a standard result when the cash-in-advance constraint applies only to consumption goods.

The first-order condition for the allocation of wealth between $m$ and $b$ is the following:

$$f_m/f_b = (A + \pi)/A \ . \tag{10}$$

Consumers substitute bonds for money in transactions as inflation rises. The determination of the ratio of money to consumption is given by (3), which can be rewritten as

$$f(m/c,\ b/c) = 1 \ . \tag{11}$$

One convenient parameterization of $f$ for discussing the sensitivity of money demand to inflation is the Constant Elasticity of Substitution (CES) function:

$$f(m,\ b) = \omega[\psi m^n + (1 - \psi)b^n]^{1/n} \tag{12}$$

where bonds and money have elasticity of substitution $\xi = 1/(n - 1)$ in transactions.

Combine (10) and (12) to obtain the following equilibrium ratio of bonds to money:

$$\Phi = \frac{b}{m} = \left[\left(\frac{1 - \psi}{\psi}\right)\left(\frac{A + \pi}{A}\right)\right]^{1/(1-n)} \ . \tag{13}$$

From (11)–(13), demand for money scaled to consumption is given by

$$\frac{m}{c} = \frac{1}{\omega}\,(\psi + (1 - \psi)\Phi^n)^{-(1/n)} \ . \tag{14}$$

8. Rebelo (1991) and Barro (1990) show that the following restriction on the parameters is needed to make discounted lifetime utility finite:

$$p > (1 - \sigma)A$$

Intuitively, momentary utility $U(c) = (c^{(1-\sigma)} - 1)/(1 - \sigma)$ must grow more slowly than the rate $\rho$ at which future utility is discounted. Inserting the growth of consumption from (9), calculating the growth of momentary utility $U(c)$, and comparing it to $\rho$ yields the expression shown. Note that the restriction can be compatible with positive growth. For example, a sufficient set of conditions (not necessary) for positive growth with finite utility is $\rho > 0$, $A > \rho$, and $\sigma > 1$.

From (13) and (14), it can be seen that money demand is unambiguously a negative function of inflation. The semielasticity of money demand with respect to inflation, noted as $\epsilon$, is a function of the elasticity of substitution between money and bonds in transactions:

$$\epsilon \equiv \frac{\partial \ln \left(\frac{m}{c}\right)}{\partial \pi} = -\left(\frac{1}{1-n}\right)\left(\frac{1}{(\psi/(1-\psi))\Phi^{-n}+1}\right)\left(\frac{1}{A+\pi}\right) . \quad (15)$$

The absolute value of the semielasticity with respect to inflation could increase or fall with inflation:

$$\frac{\partial|\epsilon|}{\partial\pi} = |\epsilon| \left\{\left(\frac{n}{1-n}\right)\left(\frac{\psi}{1-\psi}\right) \times \Phi^{-n}\left[\left(\frac{\psi}{1-\psi}\right)\Phi^{-n}+1\right]^{-1} - 1\right\}\left(\frac{1}{A+\pi}\right) . \quad (16)$$

The sign of (16) will depend on that of the second right-hand term. Considering (13), the sign condition is

$$\frac{\partial|\epsilon|}{\partial\pi} \gtreqless 0 \leftrightarrow \left(\frac{1-\psi}{\psi}\right)^{1/(1-n)}\left(\frac{A+\pi}{A}\right)^{n/(n-1)}\left(\frac{2n-1}{1-n}\right) \gtreqless 1 . \quad (17)$$

A sufficient (although not necessary) condition for $|\epsilon|$ to decrease with inflation is that the elasticity of substitution between money and bonds be smaller than two in absolute value ($|\xi| < 2$, that is, $n < 1/2$). A necessary (although not sufficient) condition for $|\epsilon|$ to increase with inflation is that $|\xi| > 2$ (that is, $n > 1/2$). The Cagan constant semielasticity could be seen as a local approximation around the inflation rate that happened to satisfy (17) with equality; however, the condition for constant semielasticity would be violated at other rates of inflation.[9]

The necessary condition for the semielasticity to rise with inflation allows us to draw a tight correspondence between empirical results on the functional form of money demand and the deep parameter $|\xi|$ which determines substitutability of money and bonds. *A rising semielasticity indicates a strikingly high elasticity of substitution between money and bonds.*

It is of interest to see how this affects the calculation of the seigniorage-maximizing rate. Seigniorage is determined by money growth, which is equal in steady state to inflation plus growth, times money holdings (scaled to consumption):

$$s = (\pi + g)\frac{m}{c} . \quad (18)$$

9. The model has been presented here in continuous time for simplicity. The discrete-time results—relevant for empirical implementation in the following section—are identical except that the continuous-time inflation rate $\pi$ should be replaced with $\pi/(1+\pi)$, where $\pi$ is the discrete-time inflation rate.

The seigniorage-maximizing inflation rate ($\pi^*$) does not have a closed-form solution. Its implicit equation is the following:[10]

$$\left(\frac{1}{1-n}\right)\left(\frac{g+\pi^*}{A+\pi^*}\right) - 1 = \left(\frac{\psi}{1-\psi}\right)^{1/(1-n)}\left(\frac{A}{A+\pi^*}\right)^{n/(1-n)} . \quad (19)$$

$\pi^*$ will be an interior maximum for seigniorage if money demand falls off more quickly than inflation rises, which requires[11]

$$\left(\frac{g+\pi^*}{|\epsilon|}\right)\frac{\partial|\epsilon|}{\partial\pi} + 1 > 0 . \quad (20)$$

The implication of this section is that Cagan's constant semielastic money demand is inconsistent with fairly general conditions of intertemporal resource and intratemporal portfolio allocation by an optimizing consumer-producer who faces cash-in-advance constraints in consumption. At an intuitive level, the higher is the elasticity of substitution between money and bonds, the lower will be the seigniorage-maximizing inflation rate, and the higher will be the likelihood that the inflation semielasticity of money demand increases with inflation.

## 2. EMPIRICAL RESULTS

This section presents empirical estimations of money demand functions and of seigniorage-maximizing inflation consistent with the model derived in section 1. The results show the importance of allowing for a variable semielasticity.

For our empirical estimates, we make use of the following money demand function:

$$\ln\left(\frac{m}{y}\right) = k + \lambda\pi^{\gamma} \quad (21)$$

where $y$ is output, $\pi$ is an appropriate measure of the opportunity cost of holding money, and $k$, $\lambda$, and $\gamma$ are parameters to be estimated.

Equation (21) differs from (14) by functional form and included variables. Output is included as the relevant scale variable instead of consumption.[12] Inflation is used

10. Derived from the standard first-order condition for maximum seignorage: $\epsilon(g + \pi) = -1$.

11. Derived from the standard second-order condition: $\partial^2 s/\partial\pi^2 < 0$.

12. Although consumption is the scale factor for money in the model of the preceding section, we chose output here for various reasons. Output is of generalized use in Cagan money demand estimates. Second, output lacks measurement noise typical of consumption series and is more readily available than the latter. Anyhow, consumption and output move together in the previous model's steady state. Unitary consumption (or output) elasticity is a feature of the cash-in-advance specification which relates money linearly to consumption (or output). However, most empirical money demand estimations do not impose unitary income elasticities. Preliminary results suggest that our main conclusions are not altered when assuming nonunitary output elasticities.

as the relevant opportunity cost of holding money rather than the nominal interest rate.[13]

The nonlinear form of equation (21) has a number of desirable properties: (i) it is simple and a straightforward generalization of the Cagan function, with a variable inflation semielasticity given by $\partial \ln (m/y)/\partial\pi = \gamma\lambda\pi^{\gamma-1}$; (ii) a necessary condition for the existence of a Laffer curve with a seigniorage maximum at a positive and finite level of $\pi$ is $\lambda < 0$ and $\gamma > 0$;[14] (iii) if $\gamma > 1$ ($\gamma = 1$, $\gamma < 1$), the absolute value of the semielasticity rises (does not change, declines) with inflation; and (iv) it can be shown to be equivalent to a nongeneralized version of the Box-Cox transformation.[15]

Three alternative measures for the opportunity cost of holding money have been typically used in money demand estimations. (a) The conventional measure of inflation, defined as the percentage rate of change of a given price index $p$ for a discrete period of time ($(p_t - p_{t-1})/p_{t-1}$) is often applied. (b) A second measure—employed by Cagan (1956) and many followers—is the discrete-time change in the natural logarithm of the price level ($\ln p_t - \ln p_{t-1}$). The two preceding measures share the disadvantage of not representing the true inflation cost of holding money during a discrete period of time. (c) As suggested by Calvo and Leiderman (1992) and others, the correct discrete-time measure of the alternative cost of holding money, equivalent to the capital loss due to inflation, is given by $[(p_t - p_{t-1})/p_{t-1}]/[1 + (p_t - p_{t-1})/p_{t-1}]$ (or by $i_t/(1 + i_t)$, when the nominal interest rate $i$ constitutes the alternative cost to holding money).

Our estimations below are based on the third, correct measure of the inflation cost of holding money. However, we will also show comparative results with the incorrect measures to illustrate the sensitivity of the results to the choice of the inflation measure.

Equation (21) is estimated for a sample of eleven high-inflation countries—the universe of all countries that had inflation rates exceeding 100 percent p.a. in at least one year during the 1960–1990 period.[16] We restricted the sample to high-inflation countries for two reasons. First, the Cagan model was originally intended as a model of high- or hyperinflation—attempting to explain money demand in low-inflation countries as a sole function of the inflation rate is bound to be a rather hopeless

13. Most of the sample countries defined below had extensive interest rate controls throughout the 1960s, some lifted them during the 1970s, and others continued with interest controls during most of the 1980s. Using inflation as the opportunity cost of holding money is relevant as long as people hold other financial assets (foreign exchange) or real assets (gold, consumer durables) with rates of return strongly correlated with inflation. All these alternative asset holdings were encompassed by holdings bonds in the preceding section. Finally, using inflation offers the advantage of a direct link to seigniorage.

14. A necessary condition for attaining a seigniorage optimum at a positive and finite level of $\pi$ is: $-(\lambda\gamma) > 0$. A necessary condition for that optimum to be a maximum is the one stated in the text. Note that if $\lambda > 0$ and $\gamma < 0$, seigniorage reaches a minimum.

15. If the Box-Cox transformation is applied to the independent variable, though not to the dependent variable, we obtain $\ln m = a + b(\pi^\gamma - 1)/\gamma$, which is equivalent to $\ln m = (a - b/\gamma) + (b/\gamma)\pi^\gamma$, which is in turn equivalent to our form. By estimating our form it is possible to identify $\gamma$, $b$, and $a$.

16. They are eight Latin American economies (Argentina, Bolivia, Brazil, Chile, Mexico, Nicaragua, Peru, and Uruguay), two African countries (Ghana and Zaire), and one Middle-Eastern economy (Israel).

endeavor.[17] Second, moderate-inflation countries constitute a class of its own, with behavioral and empirical features quite different from those found in high-inflation economies.[18]

Table 1 summarizes inflation patterns in our eleven sample countries. The table's taxonomy of inflation experiences bears resemblance to the distinction between chronic inflation and hyperinflation countries, made by Pazos (1972) and applied by Végh (1992), among others. However, here the distinction, dictated by the 1960–90 sample period, is between chronic and stable inflation, chronic and moderate inflation interrupted by high-inflation episodes (including hyperinflation), and chronic and explosive price rises. The categories of low, moderate, high, and hyperinflation coincide roughly with inflation rates in the single, lower double, triple, and quadruple (or more) digit levels—the second category coinciding with Dornbusch and Fischer's (1991) "moderate inflation" range.

Uruguay is the chronic-inflation country par excellence. The unparalleled stability of its moderately high inflation rate puts it into a category of its own, with annual rates which did not fall below 10 percent or surpass 140 percent during 1960–90. Uruguay does not even present the feature, common to all other countries, of increasing inflation after 1971–72; its three episodes of inflation exceeding 100 percent took place once each decade.

A second category is comprised of six countries of typically moderate inflation, but which experienced bursts of high- or hyperinflation during the sample period. They share a remarkably similar inflation pattern. Starting from low inflation levels in the (early) 1960s, these countries slipped into moderate inflation in the mid-1960s to mid-1970s, subsequently jumping into high price instability, which was initiated in the early 1970s (Chile) to early 1980s (Mexico). Only Bolivia experienced hyperinflation as a final stage. All countries, except Zaire, successfully stabilized during the last years, reaching surprisingly similar and moderate inflation levels in the 15–30 percent range.

Chronic and explosive inflation is observed in the last four countries. There inflation is not stationary, reaching four- and five-digit levels in the late 1980s. Peru and Nicaragua suffer the more extreme inflation explosion, starting with low inflation during the 1960s (when Argentina and Brazil already had moderate inflation) and ending with four-digit inflation levels which double those of Argentina and Brazil.

*Testing the Assumption of Constant Semielasticity of Money Demand*

Our country data is annual, for 1960–90. The dependent variable is defined as the natural logarithm of the ratio of real money balances (end-of-year nominal M1[19]

17. Preliminary estimates for low-inflation countries yielded some positive $\lambda$ coefficients.

18. According to Dornbusch and Fischer (1993), moderate inflation cases seldom end in high inflation. Seigniorage plays at most a modest role in the persistence of moderate inflation, and such inflation can be reduced only at a substantial short-term cost to growth.

19. M1 is easily measured but often not the most relevant aggregate for seigniorage collection. The domestic credit component of currency plus required bank reserves (on demand and other non-M1 deposits) could be more relevant, depending on each country's monetary and financial institutions and policies.

TABLE 1

INFLATION PATTERN IN ELEVEN HIGH-INFLATION COUNTRIES (ANNUAL INFLATION RATES, %)

1. Chronic Stable Inflation

| | Moderately High |
|---|---|
| Uruguay (3) | 56 (1960–90) |

2. Chronic Moderate Inflation with High- (Hyper-) Inflation Bursts

| | Low | Moderate | High | Hyper | Moderate |
|---|---|---|---|---|---|
| Bolivia (4) | 6 (1960–70) | 22 (1971–81) | 312 (1982–83) | 4,229 (1984–85) | 25 (1986–90) |
| Chile (5) | 8 (1960–61) | 29 (1962–71) | 294 (1972–76) | — | 26 (1977–90) |
| Ghana (4) | 5 (1960–62) | 16 (1963–76) | 77 (1977–83) | — | 17 (1984–90) |
| Israel (7) | 5 (1960–69) | 29 (1970–78) | 170 (1979–85) | — | 18 (1986–90) |
| Mexico (2) | 3 (1960–72) | 22 (1973–81) | 91 (1982–87) | — | 33 (1988–90) |
| Zaire (3) | 7 (1964–65) | 22 (1966–75) | 61 (1976–90) | — | — |

3. Chronic and Explosive Inflation

| | Low | Moderate | High | Hyper |
|---|---|---|---|---|
| Argentina (14) | — | 28 (1960–74) | 234 (1975–88) | 2,593 (1989–90) |
| Brazil (9) | — | 40 (1960–80) | 170 (1981–87) | 1,435 (1988–90) |
| Nicaragua (6) | 2 (1960–72) | 23 (1973–84) | 507 (1985–86) | 5,760 (1987–90) |
| Peru (7) | 8 (1960–72) | 52 (1973–82) | 112 (1983–87) | 3,337 (1988–90) |

NOTES: 1. Annual inflation rates are geometric averages of December-to-December rates of change (conventionally measured) of the CPI.
2. Figures in parentheses after country names denote number of years with annual inflation rates higher than 100 percent.

divided by the December CPI) to real GDP. Inflation is measured as the annual variation of the CPI between the months of December of the current and preceding years,[20] and defined as consistent with the third (correct) measure of the alternative cost of holding money, (*c*).

Both individual country and combined cross-country time-series (fixed-effects panel) estimations were performed.[21] Tables 2–3 report country results and Tables 4–5 present panel estimations. Table 2 reports the results for equation (21) in levels.

We use M1 due to lack of readily available, better monetary aggregates. As long as the measure of money one uses is sufficiently highly correlated with relevant money, all we have is measurement error in the dependent variable.

20. This timing measure of inflation is consistent only with static inflation expectations. Forward-looking timing measures—such as the annual variation of the CPI between the months of December of the current and future years—yielded similar results.

21. The sample period covers at most 1960–90 and is often somewhat shorter, depending on data availability and estimation procedure.

TABLE 2

COUNTRY ESTIMATIONS IN LEVELS

$$\ln\left(\frac{m}{y}\right) = k + \lambda\pi^{\gamma}$$

| Country | $k$ | $\lambda$ | $\gamma$ | $R^2A$ | DW | Obs | $\left(\frac{p-p_{-1}}{p_{-1}}\right)_{max} \equiv \pi_{max}$ |
|---|---|---|---|---|---|---|---|
| Argentina | −1.66 | −1.69 | 1 | 0.77 | 0.83 | 30 | 145% |
| | (0.04) | (0.18) | | | | | |
| | −1.32 | −1.93 | 0.65 | 0.77 | 0.76 | 30 | 240% |
| | (0.37) | (0.37) | (0.26) | | | | |
| Bolivia | −2.63 | −0.68 | 1 | 0.36 | 0.29 | 30 | ∞ |
| | (0.07) | (0.14) | | | | | |
| | −2.59 | −0.69 | 0.81 | 0.33 | 0.29 | 30 | ∞ |
| | (0.09) | (0.17) | (0.54) | | | | |
| Brazil | −1.69 | −1.95 | 1 | 0.76 | 0.38 | 30 | 105% |
| | (0.12) | (0.22) | | | | | |
| | −2.06 | −1.96 | 2.03 | 0.82 | 0.32 | 30 | 103% |
| | (0.10) | (0.14) | (0.67) | | | | |
| Chile | −2.96 | −0.16 | 1 | −0.03 | 0.25 | 30 | ∞ |
| | (0.17) | (0.28) | | | | | |
| | −2.97 | −0.16 | 1.37 | −0.06 | 0.25 | 30 | ∞ |
| | (0.29) | (0.27) | (9.42) | | | | |
| Ghana | −1.32 | −1.52 | 1 | 0.23 | 0.25 | 31 | 192% |
| | (0.15) | (0.56) | | | | | |
| | −1.21 | −1.35 | 0.68 | 0.21 | 0.26 | 31 | ∞ |
| | (0.12) | (0.56) | (0.40) | | | | |
| Israel | −1.47 | −2.28 | 1 | 0.69 | 0.23 | 31 | 78% |
| | (0.13) | (0.21) | | | | | |
| | −1.60 | −2.55 | 1.39 | 0.68 | 0.29 | 31 | 67% |
| | (0.12) | (0.49) | (0.51) | | | | |
| Mexico | −2.12 | −1.44 | 1 | 0.58 | 0.33 | 31 | 227% |
| | (0.05) | (0.27) | | | | | |
| | −2.16 | −1.66 | 1.24 | 0.57 | 0.39 | 31 | 127% |
| | (0.05) | (0.52) | (0.55) | | | | |
| Nicaragua | −3.53 | −1.58 | 1 | 0.49 | 0.41 | 31 | 172% |
| | (0.12) | (0.55) | | | | | |
| | −3.65 | −2.75 | 9.97 | 0.87 | 0.77 | 31 | 254% |
| | (0.08) | (0.17) | (2.12) | | | | |
| Peru | −1.91 | −1.34 | 1 | 0.60 | 0.47 | 30 | 294% |
| | (0.13) | (0.30) | | | | | |
| | −2.08 | −1.65 | 2.28 | 0.72 | 0.57 | 30 | 127% |
| | | (0.11) | (0.12) | (0.54) | | | |
| Uruguay | −1.83 | −1.36 | 1 | 0.18 | 0.26 | 31 | 278% |
| | (0.21) | (0.61) | | | | | |
| | 1.99 | −4.77 | 0.08 | 0.17 | 0.22 | 31 | ∞ |
| | (66.25) | (65.72) | (1.31) | | | | |
| Zaire | −1.29 | −1.57 | 1 | 0.44 | 0.66 | 26 | 175% |
| | (0.10) | (0.21) | | | | | |
| | −5.50 | −7.79 | 0.05 | 0.46 | 0.53 | 26 | ∞ |
| | (106.8) | (106.6) | (0.67) | | | | |

NOTE: Obs is the number of observations and $\pi_{max}$ is the implied steady-state seigniorage-maximizing inflation rate conventionally measured as in alternative (a) discussed in the text. Standard errors are in parentheses.

TABLE 3

COUNTRY ESTIMATIONS IN FIRST DIFFERENCES

$$\ln\left(\frac{m}{y}\right) - \ln\left(\frac{m}{y}\right)_{-1} = \lambda(\pi^{\gamma} - \pi^{\gamma}_{-1})$$

| Country | $\lambda$ | $\gamma$ | $R^2A$ | DW | Obs | $\left(\frac{p - p_{-1}}{p_{-1}}\right)_{max} \equiv \pi_{max}$ |
|---|---|---|---|---|---|---|
| Argentina | −0.65<br>(0.15) | 1 | 0.17 | 2.07 | 29 | ∞ |
| | −0.63<br>(0.13) | 0.80<br>(0.14) | 0.19 | 2.17 | 29 | ∞ |
| Bolivia | −0.36<br>(0.05) | 1 | 0.23 | 1.53 | 29 | ∞ |
| | −0.36<br>(0.06) | 0.96<br>(0.32) | 0.20 | 1.53 | 29 | ∞ |
| Brazil | −1.92<br>(0.36) | 1 | 0.68 | 0.66 | 29 | 109% |
| | −2.13<br>(0.10) | 3.01<br>(0.44) | 0.82 | 1.35 | 29 | 117% |
| Chile | −0.56<br>(0.17) | 1 | 0.12 | 1.70 | 29 | ∞ |
| | −1.09<br>(1,008) | 0.01<br>(6.39) | −0.02 | 1.65 | 29 | ∞ |
| Ghana | −0.65<br>(0.10) | 1 | 0.36 | 1.43 | 30 | ∞ |
| | −7.28<br>(6.30) | 5.35<br>(1.59) | 0.40 | 1.78 | 30 | 102% |
| Israel | −1.32<br>(0.15) | 1 | 0.65 | 1.41 | 30 | 313% |
| | −1.34<br>(0.13) | 1.22<br>(0.24) | 0.64 | 1.43 | 30 | 202% |
| Mexico | −0.52<br>(0.09) | 1 | 0.26 | 0.82 | 30 | ∞ |
| | −0.57<br>(0.12) | 1.38<br>(0.41) | 0.24 | 0.80 | 30 | ∞ |
| Nicaragua | −0.68<br>(0.51) | 1 | 0.01 | 1.32 | 30 | ∞ |
| | −2.62<br>(0.21) | 5.80<br>(0.61) | 0.72 | 1.53 | 30 | 167% |
| Peru | −1.30<br>(0.51) | 1 | 0.34 | 2.13 | 29 | 333% |
| | −2.34<br>(0.04) | 15.10<br>(0.34) | 0.68 | 1.59 | 29 | 376% |
| Uruguay | −0.48<br>(0.17) | 1 | 0.11 | 2.55 | 30 | ∞ |
| | −12.08<br>(1,236) | 0.01<br>(11.04) | 0.10 | 2.49 | 30 | ∞ |
| Zaire | −0.58<br>(0.07) | 1 | 0.26 | 2.13 | 25 | ∞ |
| | −0.58<br>(0.11) | 0.63<br>(0.52) | 0.23 | 2.10 | 25 | ∞ |

NOTE: Obs is the number of observations and $\pi_{max}$ is the implied steady-state seigniorage-maximizing inflation rate conventionally measured as in alternative (a) discussed in the text. Standard errors are in parentheses.

TABLE 4

PANEL ESTIMATIONS: VARIOUS SPECIFICATIONS

| Model | Variable | Estimated Coefficient | Standard Error | $R^2A$ | DW | Obs | $\left(\frac{p - p_{-1}}{p_{-1}}\right)_{max} \equiv \pi_{max}$ |
|---|---|---|---|---|---|---|---|
| *Equation* (a): $\ln(m/y) = \lambda\pi^{\gamma}$ + *country dummies* | | | | | | | |
| linear | $\gamma$ | 1 | | | | | |
| | $\lambda$ | −1.420 | 0.124 | 0.79 | 0.40 | 331 | 238% |
| nonlinear | $\gamma$ | 1.586 | 0.234 | | | | |
| | $\lambda$ | −1.526 | 0.152 | 0.79 | 0.40 | 331 | 134% |
| *Equation* (b): $(\ln(m/y) - \rho_D^* \ln(m/y)_{-1}) = \lambda(\pi^{\gamma} - \rho_D^* \pi_{-1}^{\gamma})$ | | | | | | | |
| linear | $\gamma$ | 1 | | 0.29 | 1.66 | 320 | ∞ |
| | $\lambda$ | −0.760 | 0.105 | | | | |
| nonlinear | $\gamma$ | 2.275 | 0.477 | 0.33 | 1.75 | 320 | 252% |
| | $\lambda$ | −0.943 | 0.196 | | | | |
| *Equation* (c): $\ln(m/y) = \lambda\pi^{\gamma} + b\ln(m/y)_{-1}$ + *country dummies* | | | | | | | |
| linear | $\gamma$ | 1 | | 0.95 | 1.82 | 321 | ∞ (sr) |
| | $\lambda$ | −0.643 | 0.062 | | | | 42% (lr) |
| | b | 0.816 | 0.031 | | | | |
| nonlinear | $\gamma$ | 1.672 | 0.269 | 0.96 | 1.88 | 321 | 1010%(sr) |
| | $\lambda$ | −0.704 | 0.081 | | | | 51%(lr) |
| | b | 0.809 | 0.030 | | | | |
| *Equation* (d): $(\ln(m/y) - \ln(m/y)_{-1}) = \lambda(\pi^{\gamma} - \pi_{-1}^{\gamma})$ | | | | | | | |
| linear | $\gamma$ | 1 | | | | | |
| | $\lambda$ | −0.744 | 0.114 | 0.23 | 1.69 | 320 | ∞ |
| nonlinear | $\gamma$ | 2.198 | 0.542 | | | | |
| | $\lambda$ | −0.917 | 0.221 | 0.27 | 1.76 | 320 | 266% |
| *Equation* (e): $[(\ln(m/y) - \ln(m/y)_{-1}) - \rho_D^* (\ln(m/y)_{-1} - \ln(m/y)_{-2})] = \lambda[\pi^{\gamma} - \pi_{-1}^{\gamma} - \rho_D^*(\pi_{-1}^{\gamma} - \pi_{-2}^{\gamma})]$ | | | | | | | |
| linear | $\gamma$ | 1 | | 0.23 | 2.02 | 309 | ∞ |
| | $\lambda$ | −0.713 | 0.101 | | | | |
| nonlinear | $\gamma$ | 2.016 | 0.419 | 0.28 | 2.12 | 309 | 303% |
| | $\lambda$ | −0.883 | 0.181 | | | | |

NOTE. Obs is the number of observations and $\pi_{max}$ is the implied steady-state seigniorage-maximizing inflation rate conventionally measured as in alternative (a) discussed in the text.

Table 3 presents the results for equation (21) in first differences. Each table reports results for the linear, Cagan-type specification (imposing $\gamma = 1$) and our nonlinear, variable-elasticity equation (21) (estimating $\gamma$). Tables 2–4 make use of ($c$), the correct measure of the opportunity cost of holding money. We refer to this as $\pi$. Standard errors corrected for serial correlation using the Newey-West procedure and for heteroskedasticity using the White method are reported in brackets in Tables 2 and 3.[22]

Unit root tests showed that both the independent and the dependent variable are almost invariably $I(1)$.[23] A glance at the Durbin-Watson statistics in Table 2 shows

22. The reason why some of the standard errors of the individual country coefficients are enormous is simply that in those cases the estimates of the $\gamma$ coefficient are very close to zero.

23. For the sake of brevity, the complete results of the unit root tests are not reported, though they are available from the authors. We performed Dickey-Fuller tests (with constant, and with constant and

TABLE 5

PANEL ESTIMATIONS: VARIOUS SPECIFICATIONS FOR ALTERNATIVE INFLATION MEASURES

| Model | Variable | Estimated Coefficient | Standard Error | $R^2A$ | DW | Obs | $\left(\frac{p - p_{-1}}{p_{-1}}\right)_{max} \equiv \pi_{max}$ |
|---|---|---|---|---|---|---|---|
| **1. Conventional Inflation Measure** | | | | | | | |
| 1.1 Levels: | | | | | | | |
| $\ln\left(\frac{m}{y}\right) = \lambda\left(\frac{p - p_{-1}}{p_{-1}}\right)^{\gamma}$ + *country dummies* | | | | | | | |
| linear | $\gamma$ | 1 | | | | | |
| | $\lambda$ | −0.00982 | 0.0029 | 0.66 | 0.54 | 323 | 10,183% |
| nonlinear | $\gamma$ | 0.239 | 0.049 | | | | |
| | $\lambda$ | −0.827 | 0.210 | 0.78 | 0.44 | 323 | 88,310% |
| 1.2 First Differences: | | | | | | | |
| $\left[\ln\left(\frac{m}{y}\right) - \ln\left(\frac{m}{y}\right)_{-1}\right] = \lambda\left[\left(\frac{p - p_{-1}}{p_{-1}}\right)^{\gamma} - \left(\frac{p_{-1} - p_{-2}}{p_{-2}}\right)^{\gamma}\right]$ | | | | | | | |
| linear | $\gamma$ | 1 | | | | | |
| | $\lambda$ | −0.00248 | 0.0005 | 0.07 | 1.63 | 306 | 40,323% |
| nonlinear | $\gamma$ | 0.233 | 0.044 | | | | |
| | $\lambda$ | −0.401 | 0.131 | 0.26 | 1.71 | 306 | 2,620,919% |
| **2. Log-Difference Inflation** | | | | | | | |
| 2.1 Levels: | | | | | | | |
| $\ln\left(\frac{m}{y}\right) = \lambda(\ln p - \ln p_{-1})^{\gamma}$ | | | | | | | |
| linear | $\gamma$ | 1 | | | | | |
| | $\lambda$ | −0.474 | 0.042 | 0.79 | 0.42 | 331 | 725% |
| nonlinear | $\gamma$ | 0.689 | 0.094 | | | | |
| | $\lambda$ | −0.719 | 0.092 | 0.80 | 0.38 | 331 | 1,499% |
| 2.2 First Differences: | | | | | | | |
| $\left[\ln\left(\frac{m}{y}\right) - \ln\left(\frac{m}{y}\right)_{-1}\right] = \lambda[(\ln p - \ln p_{-1})^{\gamma} - (\ln p_{-1} - \ln p_{-2})^{\gamma}]$ | | | | | | | |
| linear | $\gamma$ | 1 | | | | | |
| | $\lambda$ | −0.242 | 0.056 | 0.30 | 1.74 | 320 | 6,132% |
| nonlinear | $\gamma$ | 0.741 | 0.113 | | | | |
| | $\lambda$ | −0.378 | 0.063 | 0.31 | 1.69 | 320 | 26,149% |

NOTE: Obs is the number of observations and $\pi_{max}$ is the implied steady-state seigniorage-maximizing inflation rate conventionally measured as in alternative (a) discussed in the text.

that none of the above is a cointegrating regression.[24] As pointed out by Arrau and De Gregorio (1991), this may be the consequence of financial innovation, that is, systematic shifts in the demand for money due to the introduction of monetary substitutes. The use of dummy variables to represent "regime changes," often caused by financial innovation, may help towards obtaining cointegration. In Easterly, Schmidt-Hebbel, and Mauro (1992), we report the results obtained by introducing such dummies for periods that we selected on the basis of institutional changes and other important events that are known to have occurred in the countries in the sample. Nevertheless, introducing dummies in order to capture financial innovation does not substantially alter the results.[25]

Given the difficulty involved in obtaining cointegrating equations for money demand, we estimate our equation in first-differenced form.[26] The results, shown in Table 3, suggest more robust time-series properties than those of Table 2, as reflected by the Durbin-Watson statistics.

The results suggest both the diversity of the different country experiences and the strength of our variable-elasticity approach. The importance of relaxing the constant semielasticity assumption is vindicated by the finding that the $\gamma$s differ significantly from one in four countries (Brazil, Ghana, Nicaragua, and Peru). Massive improvements in overall fit are observed under the nonlinear form as compared to the Cagan specification. The estimated $\gamma$s are very large, exceeding significantly one, both statistically and numerically. Therefore the semielasticity of money demand with respect to inflation increases with the rate of inflation in these countries. Recalling the results of section 1, *we infer that the elasticity of substitution between money and nonmonetary financial assets is strikingly high (larger than two) in this country group.*

In the remaining countries the results for the nonlinear specification do not improve upon those corresponding to the Cagan form. The $\gamma$ coefficients are not significantly different from one (but significantly larger than zero) in Argentina, Bolivia, Israel, and Mexico, although their numerical values differ by up to 38 percent (Mexico) from the unit value assumed by the linear form. In the remaining three countries (Chile, Uruguay, and Zaire), the estimated coefficients turn out to be nonsignificant under the nonlinear form, while the $\lambda$ coefficients are significant under the Cagan form.

---

trend) using the 5 percent critical value. The opportunity cost of holding money, $\pi$, was found to be $I(0)$ in Ghana and Zaire. We could not reject the null hypothesis that the first difference of the dependent variable is nonstationary in Israel and Mexico.

24. This was confirmed by formal Dickey-Fuller and augmented Dickey-Fuller tests on the series of residuals. Complete results are available upon request.

25. Arrau and De Gregorio (1991) obtain cointegration by applying a methodology introduced by Cooley and Prescott (1976). They let the intercept term in their money demand regressions be a random walk, and *define* any changes in the intercept as "financial innovation." If the dependent and independent variables are $I(1)$, it can be shown that this procedure necessarily yields stationary residuals.

26. First-differencing is admittedly not adequate for the data generation process when individual variables are nonstationary in levels (Engle and Granger 1987). In our case, however, it is highly likely that the test results were influenced not only by structural shifts (as addressed above) but also by the small sample.

The last column of Table 3 reports seigniorage-maximizing levels of inflation ($\pi_{max}$, conventionally measured), derived from the estimated coefficients.[27] In six of the countries, no Laffer curve exists; that is, seigniorage always rises with inflation. In five other countries, however, positive maximum-seigniorage inflation rates are observed, which vary between 102 percent (Ghana) and 376 percent (Peru). The $\pi_{max}$ estimates change dramatically in two countries (Ghana and Nicaragua) when lifting the restrictive $\gamma = 1$ assumption. It is also noteworthy that in the two other countries where the $\gamma$s exceed significantly one (Brazil and Peru), the $\pi_{max}$ estimates do not differ much from those obtained under the Cagan form. But for Israel, with a $\gamma$ estimate that does not differ significantly from 1, $\pi_{max}$ falls drastically, from 313 percent under the Cagan form to 202 percent under our nonlinear specification. This implies that even if the nonlinear specification results do not differ significantly from the linear case, the numerical $\pi_{max}$ estimates can differ substantially. Therefore these country results suggest how sensitive seigniorage-maximizing inflation estimates are not only to sample choice, but, most important, to specification selection.

Our next step is to perform a number of panel regressions in order to infer more generally about the specification of money demand and related seigniorage-maximizing inflation rates in high-inflation countries. The panel estimations allow for fixed effects by introducing country-specific dummies and are performed for the sample of eleven high-inflation countries during 1960–1990. Country-specific features, such as financial structure, are assumed to be entirely captured by country fixed effects. Hence the constant term $k$ in equation (21) is allowed to vary across countries, while $\lambda$ and $\gamma$ are held invariant.

Estimates are reported in Table 4 for the level and first-differenced version of equation (21). Equation (a) is an ordinary least squares regression on the levels of ln $(m/y)$ and $\pi$, which shows very high serial correlation.[28] As a way of dealing with nonstationarity, we estimate the equation in its first-differenced form, as equation (d). As in the case of most country estimations in Table 2, this procedure reduced significantly the incidence of residual correlation. Equation (b) corrects for serial correlation by running an AR1 for panel data, following the Bhargava, Franzini, and Narendranathan (1982) methodology.[29] Equation (c) is a partial adjustment version of equation (21), for which we report both the short-run $\pi_{max}$ (sr, corresponding to the short-run semielasticity) and the long-run $\pi_{max}$ (lr, corresponding to the long-run semielasticity). Equation (e) applies the Bhargava et al. (1982) correction to equation (d).

27. $\pi_{max}$, with inflation conventionally measured ($\pi = (p - p_{-1})/p_{-1}$), is obtained from the first-order condition for seigniorage maximum, $\pi_{max} / (1 + \pi_{max}) = (-\lambda\gamma)^{-(1/\gamma)}$. Hence, $\pi_{max} = (-\gamma\lambda)^{-(1/\gamma)}/[1 - (-\gamma\lambda)^{-(1/\gamma)}]$. There exists a finite $\pi_{max}$ only if $-\lambda\gamma > 1$. Otherwise, seigniorage increases monotonicaly with $\pi$ and no Laffer curve exists, hence $\pi_{max}$ is $\infty$.

28. It seems clear that the null hypothesis that the residuals from equation (a) are white noise would be rejected by the formal test described in Bhargava et al. (1982). We did not construct tables appropriate to our own sample size. Also, it should be noted that the calculation of the Durbin-Watson statistics in Tables 3 and 4 treats appropriately residual correlation in the time dimension of the panels.

29. We discuss the details of the methodology in Easterly et al. (1992).

For both levels and first-difference specifications, the $\gamma$ coefficient estimates obtained in the nonlinear versions are significantly higher than one, validating our variable semielasticity model. In fact, the results constitute strong evidence—robust to various specification alternatives—that the inflation semielasticity of money demand increases with inflation across high-inflation countries. The estimated $\lambda$ and $\gamma$ coefficients are quite similar across estimations and reach very high significance levels in all of them. The overall fit is systematically improved by estimating the nonlinear form. Under the Cagan form, Laffer-curve maxima vary widely (between 42 percent and infinite rates of inflation); in fact, there is no Laffer curve in three of the five equations. Under our nonlinear specification, however, there is always a Laffer-curve maximum at finite inflation rates. In fact, the estimates of seigniorage-maximizing inflation rates are systematically lower in the nonlinear results as compared to the Cagan estimates. Allowing for a variable inflation semielasticity changes drastically the shape of the money demand and associated Laffer curves in high-inflation countries. We may infer that in a large sample representative of high-inflation countries—encompassing both low and high semielasticity cases—higher inflation increases on average the flight away from money and toward financial assets that provide protection from inflation.

Our preferred results are the first-differenced forms presented in equations (d) and (e) (with and without the Bhargava et al. correction, respectively). While their results are quite similar, we will focus on equation (d) to ensure comparability with the country results in Table 2.

As in other equations, the estimate of the seigniorage-maximizing inflation rate changes drastically in equation (d) when lifting the $\gamma = 1$ restriction. While no Laffer curve exists under the linear (Cagan) specification, the Laffer-curve maximum is reached at an inflation rate of 266 percent when $\gamma$ is freely estimated. The estimated coefficients ($\lambda = -0.917$, $\gamma = 2.198$), at a 100 percent rate of inflation, imply a point estimate of 0.88 for the absolute value of the semielasticity of money to inflation ($|\epsilon|$). This value increases to 1.38 when the seigniorage-maximizing inflation rate of 266 percent is reached. Beyond the Laffer-curve maximum—when the representative high-inflation economy reaches the wrong side of the Laffer curve—the semielasticity continues to rise with inflation, converging to a value of 2.01 when inflation tends to infinity. Seigniorage collection reaches a maximum of 4.0 percent of GDP at an inflation of 266 percent, falling off toward 3.6 percent of GDP when inflation tends to infinity.[30]

The central implication of these results is that the linear Cagan form misrepresents money demand in high-inflation countries. Instead of being constant, inflation semielasticities of money demand on average increase with inflation. While under the Cagan form there is no seignioarge Laffer curve, the variable semielasticity approach renders a seigniorage-maximizing level of inflation of reasonable magnitude.

30. Seigniorage collection levels are calculated from the definition of seigniorage: $S = (\pi/(1 + \pi)) * \exp[k + \lambda * (\pi/(1 + \pi))**\gamma]$. The constant $k$, which is not available from the first-difference estimations, is calculated as $[\ln(m/y)]_a - \lambda * [\pi/(1 + \pi)]_a**\gamma$, where the $a$-subindexed variables denote simple estimation sample averages.

As in the earlier country results, the finding of rising semielasticity implies a strikingly high—greater than 2—elasticity of substitution between monetary and nonmonetary assets.

*Alternative Inflation Measures*

In order to put the results of Table 4 in a broader perspective, we report in Table 5 panel estimations, based on the same data sample, for two alternative inflation measures. The first alternative uses the conventional inflation measure $((p - p_{-1})/p_{-1})$, while the second is based on the first difference of the logarithm of $p$. Both are wrong measures of the inflationary cost of holding money for a discrete time period, as discussed above.

While the pattern of preferred results for each set of estimations is similar to those of Tables 2–4 based on the correct inflation measure (first differences better than level estimations, nonlinear specifications superior to linear model results), the estimated coefficients and corresponding seigniorage-maximizing inflation estimates differ dramatically from those of Table 4. This should not come as a surprise, as the three inflation measures diverge substantially for high inflation levels.

The conventional inflation measure renders a point estimate for $\gamma$—0.233—that is significantly lower than one. The corresponding Laffer curve peak reaches an implausibly high level—at an inflation of 2,620,919 percent p.a.! The implication that the semielasticity of money demand with respect to inflation declines with the latter and that no country in the sample has been on the wrong side of the Laffer curve is empirically implausible and technically wrong, due to the flawed inflation measure used in obtaining these results.

The second incorrect inflation measure—the first-differenced logarithm of the price level used by many researchers since Cagan (1956)—also yields results with implausibly high Laffer curve maxima. The $\gamma$ is higher than that from the conventional inflation measure but still is significantly smaller than one and therefore implies a falling semielasticity.

The conclusion from these results is that inferences for the inflation semielasticity of money demand and Laffer curve maxima for high-inflation countries are also very sensitive to the choice of the inflation measure—and there is only one that accurately describes the inflation tax in discrete time.

A number of assumptions we made might be relaxed in further research, if one is willing to go beyond some of the features of our model and, in particular, if more country data and higher-frequency information becomes available. In addition to lifting the assumption of unitary income elasticity both in the long and short run, one could relax the homogeneity of degree one of real money demand in prices, and add alternative returns such as the nominal interest rate, the expected return on foreign assets, or the expected return on the stock market. Furthermore, it would be interesting to test the variable-elasticity model using more refined monetary aggregates rather than M1. However, it is by no means unambiguous what the relevant measure of money is for the government's collection of seigniorage. It might also prove useful to use real consumption rather than real GDP as a scale variable. Finally, higher-frequency data could be used when sufficiently long quarterly GDP or consumption series become available.

## 3. CONCLUSIONS

We have developed a model of money demand, inflation, and seigniorage based on an optimizing agent who faces cash-in-advance constraints in consumption. Her behavior gives rise to a money demand that exhibits a variable semielasticity with regard to inflation, as opposed to the constant-elasticity form of Cagan (1956) used by a plethora of followers. We showed that the higher is the degree of substitution between money and bonds in the consumer portfolio, the higher is the likelihood that the inflation semielasticity increases with inflation and that a seigniorage-maximizing level of inflation exists.

The empirical application of the model to a 1960–1990 sample of eleven high-inflation developing countries (defined as those with 100 percent inflation in at least one year of the sample period) led to rejecting the constant-elasticity hypothesis in four countries and, in the case of the panel estimations, for the sample as a whole. The panel results imply an absolute value of the semielasticity that increases with inflation, which implies a strikingly high degree of substitution between money and financial assets in high-inflation countries. While under the Cagan form there is no seigniorage Laffer curve, the variable semielasticity approach renders a reasonable seigniorage-maximizing inflation rate: 266 percent p.a. These results, based on the correct measure of the opportunity cost of holding money, also differ markedly from those obtained when using conventional but incorrect measures of inflation.

When drawing policy conclusions, it is important to recognize that the Keynes-Olivera-Tanzi and bracket-creep effects of inflation on tax revenue and nonindexation of government expenditure make it necessary to distinguish between the seigniorage-maximizing and the revenue-maximizing inflation rates.[31] If higher inflation implies lower real tax revenue due to the Keynes-Olivera-Tanzi effect, higher tax revenue due to bracket creep or lower government expenditure due to lack of full indexation of government expenditure, the inflation rate that maximizes total revenue to the government will diverge from that which maximizes seigniorage. As a consequence, estimates of $\pi_{max}$ will differ from the revenue-maximizing inflation rate, whether the specification of money demand is linear or nonlinear.

Our empirical results on the variable semielasticity might need to be qualified with respect to the difficulty of distinguishing between nonlinearities and shifts of the money demand schedule. One could conceive of an extreme case in which the true money demand function is linear (in the sense of a constant semielasticity), while it keeps shifting due to financial innovation. Dornbusch, Sturzenegger, and Wolf (1990) provide some evidence of hysteresis in money demand: they show that,

31. See Easterly and Schmidt-Hebbel (1994) for recent evidence on Keynes-Olivera-Tanzi and bracket-creep effects of inflation on tax revenue in developing countries.

as a consequence of high inflation, new institutions arise that allow people to economize on money holdings. This constitutes a permanent shift in the money demand function, which persists even when inflation falls back to low levels. Arrau et al. (1991) provide evidence that shifts in the money demand function occur to a greater extent in countries that experience higher (and more variable) inflation. Finally, policy-induced financial innovation could cause shifts in the demand for money. However, we found that dummies for obvious financial reform events did not alter our results.

## LITERATURE CITED

Arrau, Patricio, and José De Gregorio. "Financial Innovation and Money Demand: Theory and Empirical Implementation." The World Bank, Working paper series 585, January 1991.

Arrau, Patricio, José De Gregorio, Carmen Reinhart, and Peter Wickham. "The Demand for Money in Developing Countries: Assessing the Role of Financial Innovation." The World Bank, Working paper series 721, July 1991.

Barro, Robert J. "Inflationary Finance and the Welfare Cost of Inflation." *Journal of Political Economy* 80 (1972).

———. "Government Spending in a Simple Model of Endogenous Growth." *Journal of Political Economy* 98 (1990), S103–S125.

Barro, Robert J., and David Gordon. "A Positive Theory of Monetary Policy in a Natural Rate Model." *Journal of Political Economy* 91 (1983), 353–74.

Bhargava, A., L. Franzini, and W. Narendranathan. "Serial Correlation and the Fixed Effects Model." *Review of Economic Studies* 49 (1982), 533–49.

Blanchard, Olivier, and Stanley Fischer. *Lectures on Macroeconomics*. Cambridge: MIT Press, 1990.

Bruno, Michael, and Stanley Fischer. "Seigniorage, Operating Rules and the High-Inflation Trap." *The Quarterly Journal of Economics* (May 1990).

Buiter, Willem. *Principles of Financial and Budgetary Policy.* Cambridge: MIT Press, 1990.

Cagan, Phillip. "The Monetary Dynamics of Hyperinflation." In *Studies in the Quantity Theory of Money,* edited by M. Friedman. Chicago: University of Chicago Press, 1956.

Calvo, Guillermo, and Leo Leiderman. "Optimal Inflation Tax under Precommitment: Theory and Evidence." *American Economic Review* 82 (March 1992), 179–95.

Calvo, Guillermo, and Carlos Végh. "Currency Substitution in Developing Countries: An Introduction." *Revista de Análisis Económico* 7 (June 1992), 3–27.

Cooley, Thomas F., and Edward C. Prescott. "Estimation in the Presence of Stochastic Parameter Variation." *Econometrica* 44 (1976), 167–84.

Dornbusch, Rudiger, and Stanley Fischer. "Stopping Hyperinflation Past and Present." *Weltwirtschaftliches Archiv.* 122 (1986), 1–47.

———. "Moderate Inflation." *World Bank Economic Review* 7 (January 1993), 1–44.

Dornbusch, Rudiger, Federico Sturzenegger, and Holger Wolf. "Extreme Inflation: Dynamics and Stabilization." *Brookings Papers on Economic Activity* (1990), 1–84.

Easterly, William, and Klaus Schmidt-Hebbel. "Fiscal Adjustment and Macroeconomic Performance: A Synthesis." In *Public Sector Deficits and Macroeconomic Performance*, edited by William Easterly, Carlos Alfredo Rodríquez, and Klaus Schmidt-Hebbel. Oxford University Press, 1994.

Easterly, Wiliam, Klaus Schmidt-Hebbel, and Paolo Mauro. "Money Demand and Seignorage-Maximizing Inflation." The World Bank, Working paper series 1049, November 1992.

Engle, Robert F., and Clive W.J. Granger. "Co-Integration and Error-Correction: Representation, Estimation and Testing." *Econometrica* 55 (1987), 251–76.

Kiguel, Miguel. "Budget Deficits, Stability and the Monetary Dynamics of Hyperinflation." *Journal of Money, Credit, and Banking* 21 (May 1989), 61–80.

Kiguel, Miguel, and Nissan Liviatan. "A Policy-Game Approach to the High Inflation Equilibrium." The World Bank, Manuscript, April 1991.

Lucas, Robert E., and Nancy L. Stokey. "Money and Interest in a Cash-in-Advance Economy." *Econometrica* 55 (1987), 491–513.

Pazos, Felipe. *Chronic Inflation in Latin America.* New York: Praeger Publishers, 1972.

Rebelo, Sergio. "Long-Run Policy Analysis and Long-Run Growth." *Journal of Political Economy* 99 (June 1991), 500–21.

Rodríguez, Carlos A. "Argentina: Fiscal Disequilibria Leading to Hyperinflation." In *Public Sector Deficits and Macroeconomic Performance*, edited by William Easterly, Carlos Alfredo Rodríquez, and Klaus Schmidt-Hebbel. Oxford University Press, 1994.

Sargent, Thomas J. "The End of Four Big Hyperinflations." In *Inflation,* edited by Robert E. Hall. Chicago: University of Chicago Press, 1982.

Végh, Carlos A. "Stopping High Inflation: An Analytical Overview." *International Monetary Fund Staff Papers* 39 (September 1992), 626–95.

Walsh, Carl E. "Optimal Taxation by the Monetary Authority." *National Bureau of Economic Research Working paper* 1375, June 1984.

Van Wijnbergen, Sweder. "External Debt, Inflation, and the Public Sector: Towards Fiscal Policy for Sustainable Growth." *World Bank Economic Review* 3 (September 1989), 297–320.